1992-93 School Year

Mathematics 30

Diploma Examinations Program

Bulletin

Alberta
EDUCATION

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Mathematics 30 Diploma Examination Bulletin for 1993

1993 Diploma Examinations Schedule

Date	Time
Thursday, January 28	1 to 3:30 p.m.
Monday, June 28	9 to 11:30 a.m.
Wednesday, August 18	9 to 11:30 a.m.

1993 Marking Information

The written-response portion of the Mathematics 30 diploma examinations is marked by classroom teachers.

To qualify as markers, teachers must

- be recommended by their superintendents,
- have taught the subject for two or more years,
- be currently teaching the subject, and
- have an Alberta Permanent Professional Certificate.

Student Evaluation particularly need teachers who can mark examinations written in French.

Teachers who wish to be recommended as markers for the January 1993 examination should contact their superintendents before September 30, 1992.

Teachers who wish to be recommended as markers for the June and August 1993 examinations should contact their superintendents before March 2, 1993.

The 1993 examinations will be marked on the following dates:

January 1993 Administration	February 3 to 6
June 1993 Administration	July 6 to 10
August 1993 Administration	August 20 to 21

1992–93 Field Testing and Item Writing

As the need arises for teachers to participate in field testing and item writing, letters are sent to superintendents requesting their nominations. Teachers who are interested in these activities should let their superintendents know early in the school term.

Directions for the 1993 Field Tests

The 1993 Mathematics 30 field tests will continue to include questions that require students to describe their method of problem solving and to communicate their descriptions of mathematical definitions and situations. In addition, questions that focus on students' problem-solving abilities will also be field tested. An example of such a question is:

Homer put 1 000 bacteria with a doubling period of 1 h into a growth medium. Unfortunately, the culture was contaminated by a single bacterium with a doubling period of 30 min. When will the number of contaminating bacteria be equal to the number of desired bacteria?

One possible solution:

Let P be the number of desired bacteria: then

$P = 1\,000\,(2^t)$

Let Q be the number of contaminating bacteria: then

$Q = 1(2^{2t})$

These two quantities are equal when $P = Q$ or when

$1\,000\,(2^t) = 1(2^{2t})$.

Solving for t: $t \log 2 = \log 1\,000$

t is approximately 10 h

This is only one approach to the solution; students could use many other strategies to solve this problem.

Use of Scientific Calculators on Examinations

The term *scientific calculator* includes all hand-held devices designed for mathematical computations. These scientific calculators may have graphing capabilities, built-in formulas, mathematical functions, or other programmable features. Computers or devices with a primary function of random access storage do not fit the definition of scientific calculator. (Please refer to Appendix A for the policy statement on the use of scientific calculators on diploma examinations.)

Examinations are constructed to ensure that the use of particular scientific calculators does not advantage or disadvantage individual students.

Students should be made aware of this policy as early as possible in the school term to ensure they are able to use the scientific calculator of their choice when writing the diploma examination.

Students should also be made aware of the Examination Rules, Grade 12 Diploma Examinations (see Appendix B of this bulletin), one of which states that students must not bring notes stored in electronic devices into the examination room.

Examiners' Reports

Following the administration of the January and June examinations, examiners' reports are released. These reports briefly outline the statistical data from the examination administration and provide a diagnostic overview of student performance on each examination. Examiners' reports are designed for teacher use. Student Evaluation welcomes comments from teachers about how to increase the usefulness of these reports. Please direct your comments to the Mathematics 30 examination manager, who can be reached at 427-2948.

Annual Report

Each fall, an annual report that summarizes results from the January, June, and August diploma examination administrations is released. The purpose of this report is to inform educators and the public about student achievement in relation to provincial standards.

Assessment Standards

Provincial standards help to communicate how well students need to perform to be judged as having achieved the learnings specified for Mathematics 30. According to the *Mathematics 30 Course of Studies*, student learnings refer to specific knowledge, skill, and attitude expectations. These learnings are amplified in Appendix C, the Mathematics 30 Curriculum Standards, of this bulletin. Included in Appendix C are examples of questions that students must be able to answer to demonstrate *acceptable* or *excellent* achievement.

Students who demonstrate *acceptable* achievement but not *excellent* achievement in Mathematics 30 will receive a final mark of 50% or higher. Typically, these students have gained new skills and knowledge in mathematics but can anticipate difficulties if they choose to enrol in postsecondary mathematics courses. They have demonstrated mathematical skills and knowledge in the seven content strands of the Mathematics 30 curriculum and an ability to apply a broad range of problem-solving skills to these content strands.

Students who demonstrate *excellence* will receive a final mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in postsecondary mathematics programs; they should be encouraged

to pursue careers in which they will use their talents in mathematics.

The specific statements of assessment standards that follow were written primarily to inform Mathematics 30 teachers about the extent to which students must know the Mathematics 30 content and must demonstrate the required skills to pass the examination. The examples provided are by no means exhaustive; they are intended to provide a profile of *acceptable* and *excellent* achievement.

Student Evaluation would appreciate your feedback on these statements. Please address your concerns or your suggestions for improvement to:

Assistant Director
Mathematics/Sciences
Student Evaluation Branch
Alberta Education
Box 43, 11160 Jasper Avenue
EDMONTON, Alberta
T5K 0L2 FAX: 422-4200

Problem Solving

Students in Mathematics 30 should be able to participate in and contribute towards the problem-solving process for problems within the seven content strands.[1]

The student demonstrating acceptable achievement can:

Given the solution to a problem, analyze the solution for correctness, provide the correct response, and provide possible reasons for the problem solver's errors. For example:

Menghsha examined the graph of the function $y = 3 \sin \theta$ and determined that the domain of the function was $-1 \leq \theta \leq 1$. Is Menghsha's answer correct? If not, provide the correct answer and explain Menghsha's error.

Given one method of solving a problem, solve it a second way. For example:

Jillian was asked to find the factors of $P(x) = x^3 - 9x^2 + x - 9$. On her graphing calculator, she graphed the function and determined that the factors for $P(x)$ were $(x + 3)$, $(x + 1)$, and $(x - 3)$. If Jillian was unable to graph $P(x)$, show another method that Jillian could have used to find its factors?

Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 s, draw a diagram of the situation and create a table of values showing the relationship between the height h of a rider above the ground (the Ferris wheel is 1 m above the ground) and the time t to determine the height of a rider after 6 s.

The student demonstrating excellent achievement can:

Given a problem, solve it for the specific case(s) and then provide a general solution. For example:

Given a 4-sided and a 10-sided polygon, determine the number of diagonals in each. The student can also determine a general statement about the number of diagonals in an n-sided polygon.

Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 s, develop a mathematical model that describes the relationship between the height h of a rider above the bottom (1 m above the ground) of the Ferris wheel and the time t. The student can provide a full explanation of how his or her model was developed and can suggest alternative ways of developing the model.

[1] Italicized comments give an overview of the curriculum statements found in the *Mathematics 30 Course of Studies.*

Polynomial Functions

Given any integral polynomial function of degree 3 or less, students should be able to determine its zeros, its factors, and its graph, and should be able to describe, orally and in writing, the relationship among its zeros, its factors, and its graph.

The student demonstrating acceptable achievement can:

Given $P(x) = 10x^3 + 51x^2 + 3x - 10$, determine its zeros, its factors, and its graph. The student can also describe the relationship among its zeros, its factors, and its graph.

The student demonstrating excellent achievement can:

Given $P(x) = ax^3 + bx^2 + 3$ and that the remainders are 7 and 10 when divided by $x - 2$ and $x + 1$ respectively, determine the zeros, the factors, and the graph of $P(x)$.

Trigonometric and Circular Functions

Students should be able to solve a first-degree primary trigonometric equation and describe the relationship between its root(s) and the graph of its corresponding function.

Students should also be able to demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

The student demonstrating acceptable achievement can:

Given $y = 2 \sin (\theta - \frac{1}{2})$, $0 \leq \theta < 2\pi$, determine its zeros, describe orally and in writing the relationship between its zeros and its corresponding graph, and the effect that 2 and $-\frac{1}{2}$ have on the graph of $y = \sin \theta$.

Given $\frac{\cot \theta}{\tan \theta}$, use fundamental trigonometric identities to simplify the expression and verify this simplification by substituting values for the variable and by comparing their corresponding graphs.

The student demonstrating excellent achievement can:

Given $2 - 2 \cos^2 \theta = \sin \theta$, $0 \leq \theta < 2\pi$, determine its zeros and describe orally and in writing the relationship between its zeros and the graphs of $y = 2 - 2 \cos^2 \theta$ and $y = \sin \theta$.

Statistics

Given a problem whose solution requires the analysis of statistics, students should be able to design and administer surveys, collect and organize the results of surveys, draw inferences from surveys of bivariate data and from yes/no questions, and determine the confidence with which such inferences can be made.

Given a set of normally distributed data, students should be able to describe and analyze the data using the characteristics of a normal distribution.

The student demonstrating acceptable achievement can:

Given that a local television store in a community of 18 270 families wishes to find out the number of families that own at least 2 television sets, decide on a sample size to administer a survey, design and then administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, the student can predict the number of families that own at least 2 television sets.

Given that a local television store in a community of 18 270 families wishes to find out whether the number of families that own at least 2 television sets is related to the number of children in the family, decide on a sample size to administer a survey, design and then administer the survey, collect the results, and organize the results to reflect the data

collected. Based on the results collected, the student can predict whether or not this relationship exists and, if it does, in which direction. The student can describe orally and in writing the significance of the relationship and the confidence with which inferences to the population can be made.

Given that the results of a test were normally distributed with a mean of 21 and a standard deviation of 8, and that the passing mark was set at 15, determine the percentage of students who passed the test.

The student demonstrating excellent achievement can:

Given that a local television store in a community of 18 270 families wishes to find out the number of families that own at least 2 television sets, decide on a sample size to administer a survey, design and then administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, the student can predict the number of families that own at least 2 television sets and explain, orally and in writing, the confidence with which conclusions were made.

Given that a local television store in a community of 18 270 families wishes to find out whether the number of families that own at least 2 television sets is related to the number of children in the family, decide on a sample size to administer a survey, design and then administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, the student can determine the prediction equation of the line of best fit to decide whether or not this relationship exists. The student can explain orally and in writing the confidence with which inferences to the population were made and the significance of this confidence level.

Given that the marks on an examination were normally distributed with a mean of 54 and a standard deviation of 12, adjust the original marks by raising the mean to 64 while reducing the standard deviation to 8 and leave the z-scores unchanged. The student can determine the corresponding adjusted mark for an original mark of 36.

Quadratic Relations

Students should be able to describe orally, in writing, and by modeling, the conic resulting from the intersection of a plane and a conical surface and, from the graph of a conic, the combination of values for the numerical coefficients of the general quadratic relation that defines each graph and would result in the degenerate conics. Given the locus defining a conic section and/or the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, they can identify the conic described.

The student demonstrating acceptable achievement can:

Given $2x^2 + 2y^2 + x - 3y - 25 = 0$, identify the conic described by this equation.

Given $Ax^2 + Cy^2 + Dx - Ey - 36 = 0$, identify this as a hyperbola when $AC < 0$.

Given that a conic is represented by $3x^2 + 4y^2 + 5x + Ey - 36 = 0$, where $B = 0$, describe orally or in writing what happens to the graph of this conic when 5 is changed to -4 and -36 is changed to -9.

Given a conic that is described as having an eccentricity of 2, identify this as a hyperbola and describe its locus.

Given that the locus of points such that the sum of the distances between one of the points and two fixed points is constant, identify this locus as an ellipse.

Given a description of the intersection of a plane and a concial surface, identify the conic section formed.

Given that the cutting plane approaches the vertex of the conical surface, describe orally, in writing, and by modelling, the effect on the ellipse.

Given that the eccentricity of the orbit of Halley's Comet, which has a period of 76 years, is 0.96, sketch its graph.

The student demonstrating excellent achievement can:

Describe and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, when one or more of the numerical coefficients change.

Given a description of the intersection of a plane and a conical surface, identify orally or in writing the degenerate parabola formed.

Given a degenerate conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, describe orally and in writing the conic formed.

Given the graph and the eccentricity of a conic, describe orally and in writing the changes to the graph when the eccentricity changes.

Given that the fixed point of an ellipse is moving closer to the centre of the ellipse, describe orally and in writing the effect on the eccentricity.

Given that the eccentricity of any conic is the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, describe orally and in writing the effect that changing the eccentricity has on the relative positions of the fixed line and the fixed point.

Exponential and Logarithmic Functions

Given an exponential function, students should be able to describe orally and in writing its inverse as the logarithmic function and what it means for this information in the solution of exponential equations.

The student demonstrating acceptable achievement can:

Given $f(x) = 4^{2x}$, sketch its graph, discuss its domain and range, find the zeros of its corresponding equation, and describe orally and in writing the relationship between the zeros of its equation and its graph. The student can write the inverse of $f(x) = 4^{2x}$ in logarithmic form, sketch its graph, discuss its domain and range, determine the zeros of this equation, and describe orally and in writing the relationship between the zeros of its equation and its graph.

The student demonstrating excellent achievement can:

Given the equation $\log_5(x-4) + \log_5(x-2) = 3$, find all the possible values of x, identify the domain, describe orally and in writing the relationship between the zeros of this equation and its graph, and describe orally and in writing the reasons why there are values of x that satisfy the equation but that are not permissible for the function.

Permutations and Combinations

*Students should be able to describe orally and in writing the difference between a permutation and a combination and calculate the number of permutations or combinations of **n** things taken **r** at a time.*

The student demonstrating acceptable achievement can:

Given that there are 10 musicians in the finals of a music competition, decide whether permutations or combinations should be used to calculate the number of ways that first, second, and third prizes can be awarded.

Given the binomial $(x + 2)^5$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing x^4 (2) is obtained, determine the number of terms in the expansion of $(x + 2)^5$, and describe orally and in writing the

relationship between the number of terms in the expansion and the exponent of the binomial.

The student demonstrating excellent achievement can:

Given that five people can sit at a round table, decide whether permutations or combinations should be used to determine the number of different orders in which these five can sit at the table if Jack and Jill must sit next to one another. The student can also justify the method of solution.

Given the binomial $(3x-2y)^7$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $[(3x)^6(2y)]$ is obtained, and determine the number of terms in the expansion of $(3x-2y)^7$. The student can describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial, how the number of a given term in the expansion of $(3x-2y)^7$ relates to the exponent of $(2y)$ in that term, and how the coefficients of the terms that are equidistant from the ends of the expansion of $(3x-2y)^7$ compare in terms of combinations.

Sequences and Series

Given finite arithmetic and geometric sequences, finite arithmetic series, or finite geometric series, students should be able to describe orally and in writing the differences between sequences and series; the differences between finite and infinite; determine the terms of arithmetic and geometric sequences; and determine the sums of arithmetic and geometric series.

The student demonstrating acceptable achievement can:

Given that a sequence is defined by $t_n = 5n-3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, and determine the sum of a specified number of terms.

Given that a series is defined by $\sum_{n=3}^{6}(-2)^n$, write the terms of the series, determine whether the series is arithmetic or geometric, and determine the sum of the series.

The student demonstrating excellent achievement can:

Given that a sequence is defined by $t_n = 5n-3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, determine the sum of a specified number of terms, and determine the formula for the sum of n terms.

Given an arithmetic sequence where $t_4 + t_{13} = 99$ and $t_7 = 39$, determine the first term of this sequence.

Structure of the 1993 Examinations

Each Mathematics 30 Diploma Examination is designed to reflect the core content outlined in the *Course of Studies for Mathematics 30.* The examination is limited to those expectations that can be measured by a paper and pencil test. The time allotted to write the examination is two and one-half hours.

Core Content

The core content for the 1993 Mathematics 30 diploma examinations is emphasized as follows:

Core Content[2]	*Per Cent Emphasis*[3]
Polynomial Functions	12.5
Trigonometric and Circular Functions	18.75
Statistics	18.75
Quadratic Relations	12.5
Exponential and Logarithmic Functions	12.5
Permutations and Combinations	12.5
Sequences and Series	12.5

Design

The design of the 1993 Mathematics 30 diploma examinations is as follows:

Question Format	*Number of Questions*	*Per Cent Emphasis*
Multiple Choice	42	60
Numerical Response	7	10
Written Response	4[4]	30

[2]Core content descriptions have been shortened in this table.
[3]As suggested in the Mathematics 30/33 *Interim Teacher Resource Manual*, Alberta Education Curriculum Branch, 1991, p. 23.
[4]One written-response question will be worth 10% of the examination.

The three levels[5] of Procedures, Concepts and Problem Solving are addressed throughout the examination. Each cognitive level has the following emphasis:

	Per Cent Emphasis	
Multiple Choice and Numerical Response		
Procedures	24.5	
Concepts	21	NEW
Problem Solving	24.5	
Written Response		
Procedures, Concepts, Problem Solving	30	

Each examination is built as closely as possible to these specifications.

Directions for the 1993 Examinations

The machine-scored part of the 1993 examinations consists of a multiple-choice section and a numerical-response section.

In the multiple-choice section, students are to choose the best possible answer from four alternatives.

In the numerical-response section, students are to calculate a numerical answer. As well, students are to record their answer on a separate answer sheet, usually correct to the nearest tenth or nearest hundredth. When the answer students are to record is not a decimal value (e.g., the number of people or the degree of a polynomial), students will be asked to determine what "the number of people is______" or "the degree of this polynomial is__________". If the answer can be a decimal value, then students will be asked to record their answer correct to the nearest tenth or nearest hundredth. For instance, numerical-response question 7 on the January 1992 diploma examination asked students to calculate the numerical value of the second term of an arithmetic

[5]An explanation of levels is given in Appendix D.

sequence. Students first had to calculate the common difference and then use that value to determine the value of the second term. Although the value of the second term did result in a whole number, it could have been a decimal value if the value of the common difference was a decimal value. Hence, students were asked to record their answer "correct to the nearest tenth."

The written-response section will focus on students' understanding of the process of solving a problem and will encourage students to take risks to arrive at a solution. Students will be rewarded for selecting a problem-solving strategy and for carrying through with the strategy to find a solution. To achieve *excellence*, students must be able to select a strategy, carry it through, and complete the problem. The written-response section of the examination will also focus on students' understanding of mathematical concepts and will allow for the most flexibility in gaining an understanding of students' communication and problem-solving abilities in mathematics.

In scoring the written-response section of the examinations, markers will be evaluating students work for how well they:

- understand the problem or the mathematical concept,
- correctly use the mathematics,
- use problem-solving strategies and explain their answer and procedures,
- communicate their solutions and mathematical ideas.

Above all, students should be encouraged to try to solve the problem. Even an attempt at a solution could be worth some marks. If students leave the paper blank, markers will not be able to award any marks.

Mathematics as Communication

In keeping with the expectations listed in the *Mathematics 30 Course of Studies*, the 1993 examinations will reflect mathematics as communication. The program of studies includes communication in the problem-solving expectations: "Students will be expected to read the problem thoroughly; identify and clarify key components; restate the problem, using familiar terms; ask relevant questions; document the solution process; and explain the solution in oral or written form." (From *Mathematics 30, Course of Studies*, pp. C3-4).

These expectations are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9–12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can
>
> - reflect upon and clarify their thinking about mathematical ideas and relationships;
> - formulate mathematical definitions and express generalizations discovered through investigations;
> - express mathematical ideas orally and in writing;
> - read written presentations of mathematics with understanding;
> - ask clarifying and extending questions related to mathematics they have read or heard about;
> - appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.
>
> Focus: All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas.
>
> (From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 140)

The National Council of Teachers of Mathematics describes the evaluation of mathematics as communication in the following manner:

> The assessment of students' ability to communicate mathematics should provide evidence that they can

- express mathematical ideas by speaking, writing, demonstrating, and depicting them visually;
- understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms;
- use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations.

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 214)

Beside writing to communicate results, the accuracy of and logic in students' mathematical statements also reflects mathematics as communication. Hence, students will continue to be expected to demonstrate logical and meaningful communication on the diploma examinations.

Mathematics as Problem Solving

In keeping with the expectations identified in the *Mathematics 30 Course of Studies*, the 1993 examinations will reflect mathematics as problem solving. Problem solving is integrated throughout the content areas in the curriculum. A set of specific problem-solving learner expectations is identified before the specific content learner expectations.

The expectations contained in the program of studies are consistent with the recommendations made by the National Council of Teachers of Mathematics:

In grades 9–12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

Focus: In grades 9–12, the problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the student's approach to doing mathematics, regardless of the topic at hand. From this perspective, problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics as identified in later standards is both constructed and reinforced.

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 137)

Evaluating Communication and Problem Solving

The open-ended question is a way in which to examine mathematics as communication and mathematics as problem solving. An open-ended question allows students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

Alberta Education encourages you to obtain a copy of the following two documents to examine the open-ended question in further detail:

Assessment Alternatives in Mathematics: An overview of assessment techniques that promote learning.

A Question of Thinking: A First Look at Students' Performance on Open-ended Questions in Mathematics.

Both these documents are available through:

California State Department of Education
Bureau of Publications, Sales Unit
P. O. Box 271
Sacramento, CA 95802-0271

The following document was recently published by the National Council of Teachers of Mathematics. Alberta Education encourages you to obtain a copy of it as well because it provides some good practical suggestions for the assessment of problem solving and communication on a regular basis:

Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions.

This document is available through:

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091

Exponential and Logarithmic Functions

The *Mathematics 30 Course of Studies* states that:

Students will be expected to demonstrate an understanding that many real-world phenomena exhibit exponential properties.

Students will be expected to recognize exponential functions describing situations involving exponential growth and decay.

Students will be expected to solve problems involving exponential growth and decay.

Students will be expected to demonstrate an understanding that many phenomena exhibit characteristics that can be described using logarithmic functions.

Students will be expected to recognize logarithmic functions that describe situations that have logarithmic characteristics.

Students will be expected to solve problems that exhibit logarithmic properties by developing and solving logarithmic equations.

Given these learner expectations, the diploma examinations will not necessarily provide exponential growth and decay formulas or logarithmic formulas. Because there are a variety of ways of solving a problem, students will be expected to know how these formulas are developed.

Trigonometric and Circular Functions

In past diploma examinations, structure has been added to assist students when they are required to prove trigonometric identities. That is, the diploma examination question looked like this:

Prove that $(1 + \cos\theta)(\csc\theta - \cot\theta) = \sin\theta$, where $\theta \neq n\pi$, $n \in I$.

SHOW CLEARLY ALL SUBSTITUTIONS AND PROCEDURES.

LEFT SIDE	RIGHT SIDE

Students were expected to provide their proofs within these guidelines. Consistent with students communicating their knowledge of mathematics, diploma examinations **will no longer** have these guidelines.

Quadratic Relations

In a 1992 diploma examination, the term "double-napped" cone was used when students were asked to describe the conic section formed when a plane cut a cone. The *Mathematics Dictionary*, James/James 4th Edition, states:

These are called conics, or **conic sections**, since they can always be gotten by taken plane sections of a conical surface. (p. 71)

A **conical surface** is defined as a

surface which is the union of all lines that pass through a fixed point and intersect a fixed curve. The fixed point is the **vertex**, or **apex**, of the conical surface,

the curve the **directrix**, and each of the lines is a **generator** or **generatrix**. (p. 72)

A circular conical surface is defined as a conical surface whose directrix is a circle and whose vertex is on the line perpendicular to the plane of the circle. (p. 72)

Note: The directrix used in the definition of the conical surface is not the same directrix used in the definition of eccentricity.

The circular conical surface is the "double-napped" cone described in the *Mathematics 30 Teacher Resource Manual*. James/James further uses the circular conical surface in defining the ellipse and hyperbola.

As this dictionary is an authorized resource and the *Mathematics 30 Teacher Resource Manual* is available for all provincial Mathematics 30 teachers, the diploma examinations will reflect these definitions.

Statistics

In marking written-response question 3 on the June 1992 diploma examination, markers discovered that some students confused **proportion of sample** with **percentage of population** and therefore misinterpreted the *Charts of 90% Box Plots*.

For example, if a student surveys a **sample** of size 40, and 36 people say "yes". Using the *Charts of 90% Box Plots*, the student is 90% confident that between 80% and 95% of the population will also say "yes". On the June 1992 examination, however, a number of students translated the 36 of 40 people as being 90% **of the population** and then read the width of the box plot as the confidence interval. If a student does read the width of the box plot to be the confidence interval, then the confidence interval for this survey would be between approximately 82% and 98%.

Students should also be aware that when they read the *Charts of 90% Box Plots* to determine the true confidence interval, they may have to interpolate. For example, the results of a survey of 40 people indicate that 24 people say "yes" to the question. On the *Charts of 90% Box Plots*, the lower limit is between 45% and 50%, and the upper limit is between 70% and 75%. In this case, students must estimate the true confidence interval as approximately 47%–72%. This confidence interval indicates that the surveyors are 90% confident that between 47% and 72% of the population will respond "yes" on the same survey. When the narrower confidence interval of 50%–70% is reported, the surveyors are implying that they are 90% confident of their results, whereas the true probability of this interval is less than 90%.

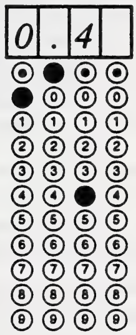
The Answer Sheet

There will continue to be a common answer sheet for the machine-scored part of the 1993 series of diploma examinations for mathematics, chemistry, and physics. All three subjects will use common instructions and a common form for the numerical-response questions. The format of the answer sheet allows students to place the decimal point in an appropriate position. In **all** cases, students will be required to fill in the answer beginning at the left field and leave any unused fields blank. See Appendix E for an explanation of significant digits and rounding.

The following examples illustrate the use of the answer sheet.

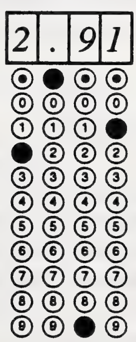
Example 1: If $\csc\theta = 2.6$, $\frac{\pi}{2} < \theta < \pi$, then the value of $\sin\theta$ correct to the nearest tenth is______________.

Value:	0.3846. . .
Value to be recorded:	0.4



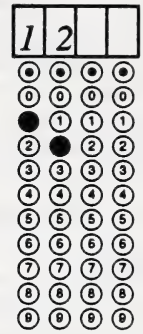
Example 2: The sketch of the graph of $y = \log_2(x)$ is shown below. If $P(x, 1.54)$ is a point on this graph, then the value of x correct to the nearest hundredth is__________.

Value:	2.9079. . .
Value to be recorded:	2.91



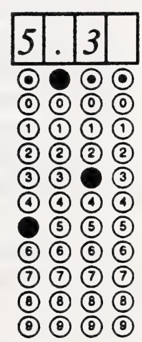
Example 3: If $x^{10} - 8x + 3$ is divided by $x + 1$, then the remainder is__________.

Value:	12
Value to be recorded:	12



Example 4: A university biology class consists of 200 students. The marks of the final examination are normally distributed with a mean mark of 45.0 and a standard deviation of 5.3. The professor adjusts the marks by adding 5.0 to each grade. Correct to the nearest tenth, the standard deviation of the adjusted marks is ____________.

Value:	5.3
Value to be recorded:	5.3



Appendix A

Policy: Use of Calculators on Alberta Education Diploma Examinations

Background

The knowledge, skills and attitudes relevant to technology and its uses are being incorporated into courses and programs of study wherever appropriate. Students are expected to learn the advantages and limitations of technological developments and their impact upon society. The ability to use technology helps students understand and appreciate the process of technological change, gives added depth to programs, and provides the basis for the development of skills and understanding. These expectations are reflected in the diploma examinations. Since the data provided for writing diploma examinations in mathematics, chemistry, and physics does not include information such as logarithms and trigonometric functions, students will need to use scientific calculators for these exams.

Definition

This policy considers a scientific calculator to be a hand-held device designed primarily for mathematical computations. Included in this definition are those scientific calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features.

Policy

To ensure compatibility with provincial *Programs of Study* and equity and fairness for all students, Alberta Education expects students to use scientific calculators, as defined above, when they are writing diploma examinations in mathematics, chemistry, and physics. Examinations are constructed to ensure that the use of particular models of calculators neither advantages nor disadvantages individual students.

Procedures

1. At the beginning of a course, teachers must advise students of the calculators that may be used when they are writing mathematics, chemistry, and physics diploma examinations.
2. In preparation for calculator failure, students may bring extra calculators and batteries into the exam room.
3. During exams, supervising teachers must ensure that
   a. all calculators fall within the definition provided with this policy,
   b. all calculators operate in silent mode,
   c. students do not share calculators,
   d. students do not bring external devices to support calculators into the examination room. Such devices include manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, or any annotations outlining operational procedures for scientific calculators.

Appendix B

Examination Rules, Grade 12 Diploma Examinations

1. **Admittance to the Examination Room**
   Students must not enter or leave the examination room without the consent of the supervising teacher.
2. **Student Identification**
   Students must present identification that includes their signature and photograph. One of the following documents is acceptable: driver's licence, passport, or student identification card. Students must not write an examination under a false identity or knowingly provide false information on an application form.
3. **Identification on Examinations**
   Students must not write their names or the name of their school anywhere in or on the examination booklet other than on the back cover.
4. **Time**
   Students must write an examination during the specified time and may not hand in a paper until at least one hour of the examination time has elapsed. Students who arrive more than one hour after an examination has started will not be allowed to write the examination. Students who arrive late but within the first hour of an examination sitting may be allowed to write only at the discretion of the supervising teacher.
5. **Discussion**
   Students must not discuss the examination with the supervising teacher unless the examination is incomplete or illegible. Students must not talk, whisper, or exchange signs with one another.
6. **Answer Sheets**
   Students must use an HB pencil to record their answers on the machine-scorable answer sheets.
7. **Written Responses**
   All work for the written-response sections of the diploma examinations must be done in the examination booklet. Students are expected to write their revised work in blue or black ink for English 30, English 33, Français 30, Social Studies 30, and Biology 30.
8. **Material Exchanges**
   Students must not copy from other students or exchange material. Notes in any form—including those on papers, in books, or stored in electronic devices—must not be brought into the examination room. Calculator programs designed to perform mathematical computations or those designed to assist students in graphing are not classified as notes.
9. **Materials Allowed**
   English 30, English 33: Students may use a dictionary and a thesaurus for Part A only. Electronic devices are not allowed for either part.
   Français 30: Students may use a dictionary, a thesaurus, and a book of verb forms for Partie A only. Electronic devices are not allowed for either part.
   Social Studies 30, Biology 30: Students may not use electronic devices.
   Mathematics 30: Tear-out data pages are provided in the examination booklet. Students may use scientific calculators (see *Calculator Policy, General Information Bulletin*) but must not share them.
   Chemistry 30, Physics 30: A separate data booklet is provided for each of these examinations. Students may use scientific calculators (see *Calculator Policy, General Information Bulletin*) but must not share them.

   Students are expected to provide their own writing materials, including pens and HB pencils, calculators, or other necessary instruments. Tear-out pages for rough work are provided in each biology, chemistry, mathematics, and physics examination booklet.
10. **Translation Dictionaries**
    Students are not allowed to use translation dictionaries in any subject. Exchange students must satisfy the same requirements as other students.

Appendix C

Mathematics 30 Curriculum Standards

The Curriculum Standards provided in this appendix are intended to clarify the *Mathematics 30 Course of Studies* statements. Included are examples of questions that students must be able to do to demonstrate *acceptable* or *excellent* achievement. For a definition of *acceptable* and *excellent* achievement, see page 2.

Problem Solving

Students in Mathematics 30 can participate in and contribute towards the problem-solving process for problems within the seven content strands.[1]

Polynomial Functions

Given any integral polynomial function of degree 3 or less, students can determine its zeros, its factors, and its graphs and can describe, in writing, the relationship among its zeros, its factors, and its graphs.

Students demonstrating acceptable achievement can:

- recognize and give examples of polynomial function OF DIFFERENT DEGREES[2];
- generate the graph of any integral polynomial function with the use of graphing calculators or graphing utility packages;
- use the Remainder Theorem to evaluate a THIRD-DEGREE INTEGRAL polynomial function for rational values of the variable and to understand how this can be used to find factors of the polynomial function;
- factor and find the zeros for an integral polynomial function in standard form, degree 3 or less, in which all zeros are rational;
- find approximations for all the real zeros of integral polynomial functions using graphing calculators or computers;
- derive an equation of an integral third-degree polynomial function given its rational zeros;
- recognize the general shape of graphs of integral polynomial functions of degree 4 or less where the multiplicity of zeros is one, two, OR THREE;
- IDENTIFY THE POTENTIAL RATIONAL ZEROS OF AN INTEGRAL POLYNOMIAL FUNCTION;
- DETERMINE THE MINIMUM DEGREE OF A POLYNOMIAL FUNCTION BY USING THE MULTIPLICITIES OF ITS ZEROS;
- participate in and contribute toward the problem-solving process for problems that can be represented by polynomial functions studied in Mathematics 30.

Students demonstrating excellent achievement can also:

- use the Remainder Theorem when either the factor or the original polynomial contains unknown coefficients;
- explain the relationships between the graphs of different polynomial functions and their zeros;
- derive an equation for an integral polynomial function given its zeros and any other information that will uniquely define it;
- USE THE REMAINDER THEOREM TO EVALUATE INTEGRAL POLYNOMIAL FUNCTION BEYOND THE THIRD DEGREE FOR RATIONAL VALUES OF THE VARIABLE AND UNDERSTAND HOW THIS CAN BE USED TO FIND FACTORS OF THE POLYNOMIAL FUNCTION;
- RECOGNIZE THE GENERAL SHAPE OF GRAPHS OF INTEGRAL POLYNOMIAL FUNCTIONS OF DEGREE *N* WHERE THE MULTIPLICITY OF ZEROS IS GREATER THAN TWO;
- complete the solution to problems that can be represented by polynomial functions studied in Mathematics 30.

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[1] Italicized comments give an overview of the curriculum statements found in the *Mathematics 30 Course of Studies.*

[2] Words in capitalized format are clarifications or additions to the 1991-92 Mathematics 30 bulletin.

The student demonstrating an acceptable achievement can do the following types of questions, as can the student demonstrating excellent achievement.

1. One factor of $10x^3 + 51x^2 + 3x - 10$ is $x + 5$. The other two factors are

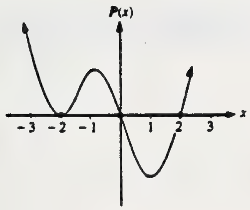
* A. $2x + 1$ and $5x - 2$
B. $2x - 1$ and $5x + 2$
C. $2x + 5$ and $5x - 1$
D. $2x - 5$ and $5x - 1$

2. For an integral polynomial function $P(x)$, $P(5) = 0$ and $P(-2) = 0$. One factor of this polynomial is

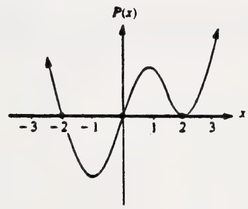
A. $x - 2$
B. $x + 5$
* C. $x^2 - 3x - 10$
D. $x^2 + 3x - 10$

3. The sketch that illustrates the graph of $P(x) = ax(x + 2)(x - 2)^2$, where $a > 0$, is

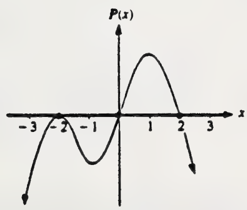
A.



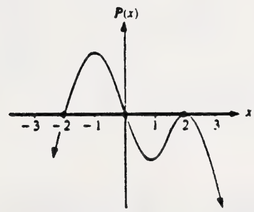
*B.



C.



D.



The student demonstrating excellent achievement can also identify the sketch that illustrates the graph of $P(x) = -ax(x + 2)(x - 2)^2$, where $a > 0$.

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*Correct answer.

4. When $5x^3 - 7x^2 + 2x + 1$ is divided by $x - 3$, the remainder correct to the nearest tenth is [79.0]*

The student demonstrating excellent achievement can do questions such as:

1. When $ax^2 + bx + 5$ is divided by $x - 2$, the remainder is 7, and when divided by $x + 1$, the remainder is 10. The value of b is

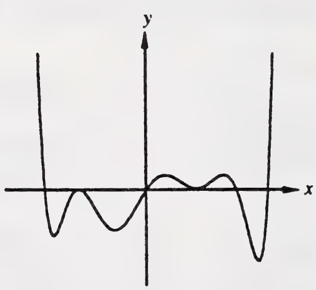
   A. 3
   B. 2
   C. −2
 * D. −3

2. If −1 and −2 are x-intercepts of the graph of $y = x^3 + ax^2 - x + b$, then the values of a and b respectively are

   A. 2 and 2
 * B. 2 and −2
   C. −2 and 2
   D. −2 and −2

3. A calculator display showed the graph at the right. The lowest degree of the polynomial represented by the graph is

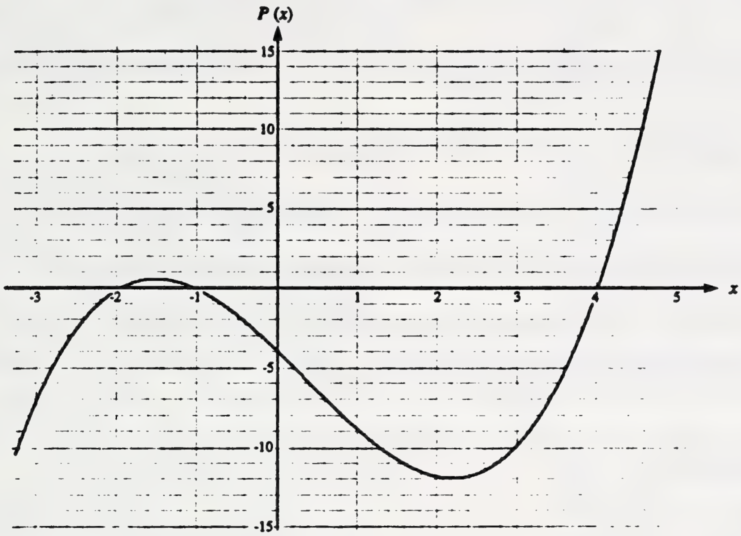
   A. 5
   B. 6
   C. 7
 * D. 8



4. If $2x^3 + kx^2 - mx - 5$ is divided by $x - 3$, the remainder is 6. The equation relating k and m is

   A. $9k - 3m = -49$
 * B. $9k - 3m = -43$
   C. $9k + 3m = 59$
   D. $9k + 3m = 65$

5. The graph of $P(x) = a(x-p)(x-q)(x-r)$, as represented below, is displayed on a computer screen. Pat's assignment is to find the value of a. From the graph, Pat notes that the x-intercepts are -1, -2, and 4.



Pat finds that the value of a is

A. $\frac{3}{4}$

* B. $\frac{1}{2}$

C. -4

D. -10

6. If $x-c$ is a factor of $6x^3 + 3cx^2 - c^2x - 27$, then the value of c correct to the nearest tenth is_________[1.5]*.

DRAFT

The student demonstrating an acceptable achievement can successfully *complete part a* of the following three-part question, whereas the student demonstrating excellent achievement can successfully *complete all three parts* of this three-part question.

7. The graph of a third-degree polynomial function touches the x-axis at (1, 0) and crosses the x-axis at(–2, 0). Express in factored form

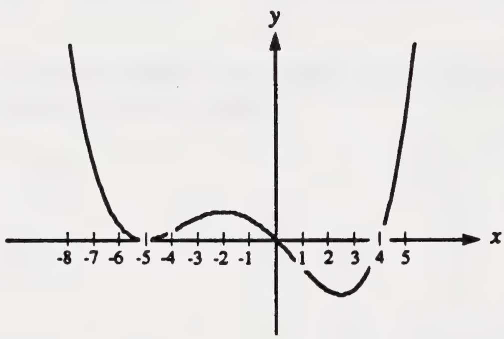
   a. an equation for such a polynomial function

   b. the equation of the polynomial function if the y-intercept of its graph is –6

   c. the equation of the polynomial function if its graph passes through (2, 8)

The student demonstrating an acceptable achievement can successfully *complete parts a and b* of the following three-part question, whereas the student demonstrating excellent achievement can successfully *complete all three parts.*

8. This is a partial sketch of the graph of a polynomial function with domain $-8 \leq x \leq 5$. There are no other x-intercepts.



   a. What are the zeros of this polynomial?

   b. What is the lowest possible degree of this polynomial?

   c. What other degrees could this polynomial be? Explain your answer.

Trigonometric and Circular Functions

Students can solve a first-degree primary trigonometric equation and describe the relationship between its root(s) and the graph of its corresponding function.

Students can also demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

Students demonstrating acceptable achievement can:

- convert angle measurements between degree and radian measure;
- GIVEN ANY TWO OF THE FOLLOWING MEASUREMENTS—THE RADIAN MEASURE OF THE CENTRAL ANGLE, THE RADIUS, OR THE LENGTH OF AN ARC—DETERMINE THE UNKNOWN MEASUREMENT;
- verify[3] the fundamental trigonometric identities;
- solve first-degree trigonometric equations on the domain $0 \leq \theta < 2\pi$ IN RADIANS AND $0° \leq \theta < 360°$;
- simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities;
- generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages;
- explain the effect of EACH PARAMETER a, b, c, and d on the graph of the $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$ functions;
- state the domain and range of $y = \sin \theta$, $y = \cos \theta$, and $y = \tan \theta$;
- describe, orally and in writing, the relationship between the root(s) of a FIRST-DEGREE trigonometric equation and the graph of its corresponding function;
- participate in and contribute toward the problem-solving process for problems that can be represented by trigonometric functions studied in Mathematics 30.

Students demonstrating excellent achievement can also:

- prove[4] trigonometric identities;
- explain, orally and in writing, the COMBINED EFFECTS of the parameters a, b, c, and d in the trigonometric functions $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$, on the functions' domain and range;
- solve first-and second-degree trigonometric equations INCLUDING DOUBLE AND HALF angles on the domain $0 \leq \theta < 2\pi$ AND $0° \leq \theta < 360°$;
- DESCRIBE, ORALLY AND IN WRITING, THE RELATIONSHIP BETWEEN THE ROOT(S) OF A TRIGONOMETRIC EQUATION AND THE GRAPH OF ITS CORRESPONDING FUNCTION;
- complete the solution to problems that can be represented by trigonometric functions studied in Mathematics 30.

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[3]For a definition of verify, see the *Mathematics 30 Course of Studies*, p. 6.

[4]For a definition of prove, see the *Mathematics 30 Course of Studies*, p. 6.

The student demonstrating an acceptable achievement can do the following types of questions, as can the student demonstrating excellent achievement.

1. Correct to the nearest tenth of a radian, an angle of 105° is

* A. 1.8 rad
  B. 2.4 rad
  C. 4.0 rad
  D. 5.4 rad

2. The expression $\frac{\cot\theta}{\tan\theta}$ is equivalent to

  A. $\frac{\cos\theta}{\sin\theta}$

  B. $\frac{\sin\theta}{\cos\theta}$

  C. $\frac{\sin^2\theta}{\cos^2\theta}$

* D. $\frac{\cos^2\theta}{\sin^2\theta}$

3. If the graph of $y = \sin\theta$ undergoes a phase shift of $\frac{\pi}{2}$ radians to the right and an amplitude increase to π, then the equation of the resulting graph is

  A. $y = \frac{\pi}{2}\sin(\theta + \pi)$

  B. $y = \frac{\pi}{2}\sin(\theta - \pi)$

* C. $y = \pi\sin\left(\theta - \frac{\pi}{2}\right)$

  D. $y = \pi\sin\left(\theta + \frac{\pi}{2}\right)$

4. The expression $\frac{1 - \sin^2\theta}{\sin^2\theta}$, where $\theta \neq n\pi$, $n \in I$, is equivalent to

* A. $\cot^2\theta$
  B. $\tan^2\theta$
  C. 1
  D. 0

5. If $\sin\theta = \frac{5}{13}$, $\frac{\pi}{2} < \theta < \pi$, then the value of $\csc\theta$ correct to the nearest tenth is _____[2.6]*.

DRAFT

The student demonstrating excellent achievement can do questions such as:

1. The expression $\dfrac{\sec\theta\sin\theta}{\csc\theta\cos\theta}$ is equivalent to

• A. $\tan^2\theta$
B. $\cot^2\theta$
C. $\sin\theta\cos\theta$
D. $\sin^2\theta\cos^2\theta$

2. If $2-2\cos^2\theta=\sin\theta$, $0\le\theta<2\pi$, then all possible values of θ are

A. $0, \frac{\pi}{2}, \pi$

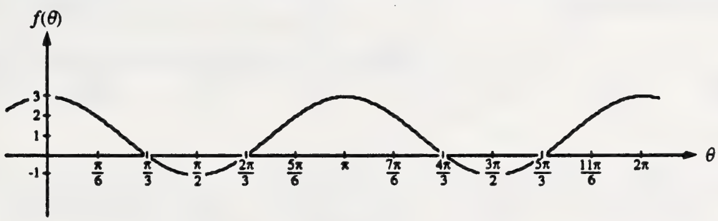
• B. $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

C. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

D. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

3. If the solutions to $A\sin^2\theta-B\sin\theta+1=0$, $0°<\theta\le 90°$ are 30° and 90°, then the value of B correct to the nearest tenth is ______[3.0]*.

4. The graph of $f(\theta)=2\cos(2\theta)+1$, as represented below, is displayed on a computer screen.



Kelly is asked to find all the values of θ that satisfy the equation $2\cos(2\theta)=-1$, $0\le\theta\le 2\pi$. Kelly finds that all the values of θ that satisfy this equation are

A. $\frac{\pi}{2}, \frac{3\pi}{2}$

B. $\frac{2\pi}{3}, \frac{4\pi}{3}$

C. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

• D. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

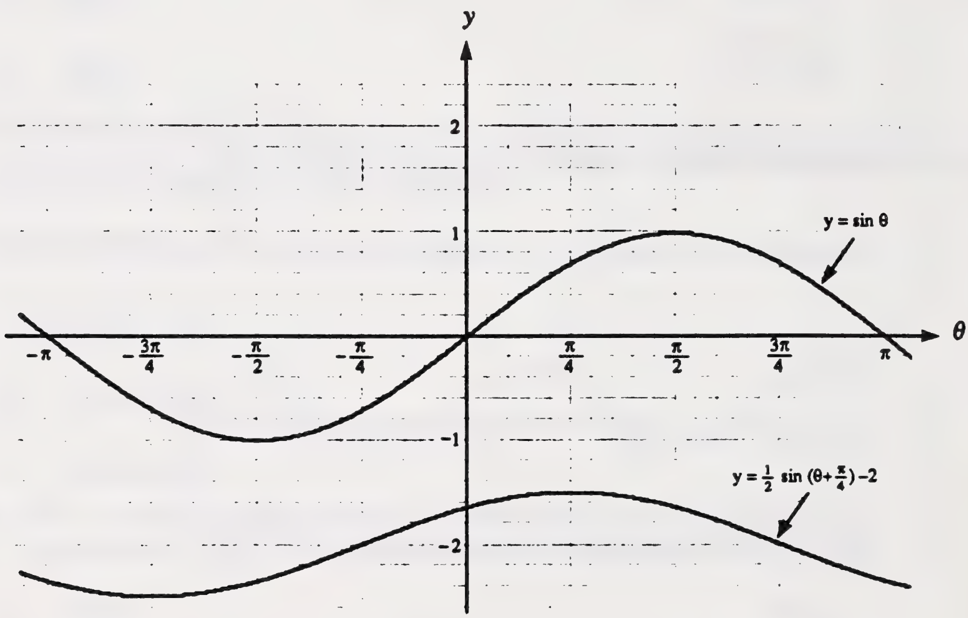
The student demonstrating acceptable achievement can successfully *complete the initial substitutions for csc θ and cot θ* in this written-response question, whereas the student demonstrating excellent achievement can successfully *complete* the proof.

5. Prove that $(1 + \cos\theta)(\csc\theta - \cot\theta) = \sin\theta$, where $\theta \neq n\pi, n \in I$.

   SHOW CLEARLY ALL SUBSTITUTIONS AND PROCEDURES.

The student demonstrating acceptable achievement can successfully *describe the individual effects of the parameters* $\frac{1}{2}$, $+\frac{\pi}{4}$, and −2 on the graph of $y = \sin\theta$ in this written-response question, whereas the student demonstrating excellent achievement can also *describe the effects of these parameters on the domain or range* of the function.

6. You are helping your friend analyze the graphs of trigonometric functions. Your friend wants to know the effects of the parameters a, b, c, and d in $y = a \sin b(\theta + c) + d$. You start by graphing $y = \sin\theta$ and $y = \frac{1}{2}\sin(\theta + \frac{\pi}{4}) - 2$ as shown below.



Describe the effects of the parameters $\frac{1}{2}$, $+\frac{\pi}{4}$, and −2 on the graph of $y = \sin\theta$.

DRAFT

Statistics

Students can design AND administer SURVEYS, collect AND organize THE RESULTS OF SURVEYS, draw inferences from surveys OF BIVARIATE DATA AND FROM YES/NO QUESTIONS, AND DETERMINE THE CONFIDENCE WITH WHICH SUCH INFERENCES CAN BE MADE.

Students can also describe and analyze data using the characteristics of a normal distribution.

Students demonstrating acceptable achievement can:

- collect and plot bivariate data;
- design AND administer SURVEYS, collect AND organize results, and draw inferences from surveys OF BIVARIATE DATA AND FROM YES/NO QUESTIONS AND DETERMINE THE CONFIDENCE WITH WHICH SUCH INFERENCES CAN BE MADE;
- ASSESS THE STRENGTHS, WEAKNESSES, AND BIASES OF SAMPLES;
- recognize and describe the apparent correlation between the variables of a bivariate distribution from a scatter plot;
- plot a line of best fit on a scatter plot using the median fit method;
- use charts of 90 % box plots to find the confidence interval within which survey results can be interpreted;
- DESCRIBE, ORALLY AND IN WRITING, WHAT IS MEANT BY A CONFIDENCE INTERVAL;
- FIND AND interpret the mean and standard deviation of a set of normally distributed data;
- ANALYZE THE RESULTS OF SURVEYS, INCLUDING MAKING INFERENCES TO THE POPULATION AND EVALUATING THE RESULTS FOR THE CONFIDENCE WITH WHICH THEY MAY BE HELD;
- apply the standard normal curve and the z-scores of data that are normally distributed;
- participate in and contribute toward the problem-solving process for problems that require the analysis of statistics studied in Mathematics 30.

Students demonstrating excellent achievement can also:

- develop and use prediction equations of the line of best fit to make inferences for populations;
- draw statistical conclusions, make inferences to populations and explain the confidence with which conclusions and inferences are made based on the results of surveys;
- complete the solution to problems that require the analysis of statistics studied in Mathematics 30.

The student demonstrating an acceptable achievement can do the following types of questions, as can the student demonstrating excellent achievement.

1. The results of a test were normally distributed with a mean of 21 and a standard deviation of 8. If the passing mark was set at 15, then the percentage of students who passed the test was

   A. 84.38%
   B. 81.50%
   * C. 77.34%
   D. 72.66%

2. A mark of 73 on an examination translates to a z-score of 1.6. If the mean is 64, then the standard deviation correct to the nearest tenth is____[5.6]*.

3. The length of the confidence intervals increases as the

* A. sample size decreases
  B. sample size increases
  C. length of the questionnaire increases
  D. length of the questionnaire decreases

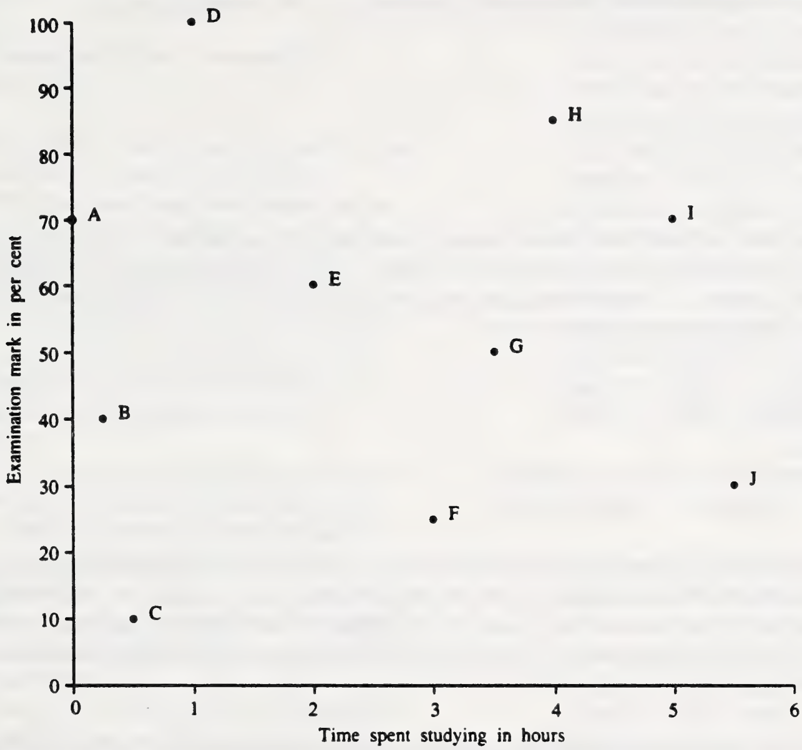
4. In a community of 18 270 families, 90 families were surveyed regarding the number of television sets they own. The results are summarized in the table below.

Survey Results	
Number of TV Sets	Number of Families
0	4
1	38
2	45
3	3

Based on these survey results, the expected number of families owning at least two television sets is

  A. 8222
  B. 8770
  C. 9135
* D. 9744

5. The examination mark received by each of 10 students and the amount of time each student spent studying are shown on the scatter plot below. The students are represented by the letters A to J.



Using the information obtained from the scatter plot, one could say that the time spent studying

* A. correlates not obviously with the examination mark received
B. correlates positively with the examination mark received
C. correlates strongly and positively with the examination mark received
D. correlates negatively with the examination mark received

The student demonstrating excellent achievement can do questions such as:

1. The mean on a test is $5k$ with a standard deviation of $k - 2$. A student's score on the test is represented by $8k - 16$. If the student's z-score is 2, then the actual score is

* A. 80
B. 60
C. 20
D. 10

2. The marks on an examination were normally distributed with a mean of 54 and a standard deviation of 12. A decision was made to adjust the original marks by raising the mean to 64 while reducing the standard deviation to 8 and leaving the z-scores unchanged. For an original mark of 36, the corresponding adjusted mark would be

A. 42
B. 46
* C. 52
D. 56

The student demonstrating acceptable achievement can *start the solution* to this written-response question by drawing a diagram or by determining the number of cases that contain more than 39 apples or the number of cases that contain less than 66 apples. The student demonstrating excellent achievement *can determine* the number of cases that contain between 39 and 66 apples.

3. Cases of apples chosen at random contain a mean of 48 apples per case with a standard deviation of 10. If the number of apples per case is distributed normally, and a wholesaler purchases 800 cases, how many cases would be expected to contain between 39 and 66 apples?

The student demonstrating acceptable achievement as well as the student demonstrating excellent achievement can *successfully complete all three parts* in this written-response question.

4. Forty bus commuters were asked if they believed their bus service was adequate. Sixteen of the 40 commuters answered "yes".

   a. Using the "90% Box Plots from Samples of Size 40" tear-out page near the end of the booklet, determine the 90% confidence interval for the percentage of yeses in the population.

   b. Describe what is meant by this 90% confidence interval.

   c. How would this confidence interval change if the sample size increased?

Quadratic Relations

Students can describe the conditions that generate conic sections.

Students demonstrating acceptable achievement can:

- describe orally, in writing, and by modeling, the intersection of a plane and a CONICAL SURFACE that would result in a hyperbola, an ellipse, a parabola, and a circle;
- DESCRIBE, ORALLY, IN WRITING, AND BY MODELING, AND IDENTIFY THE POSITION OF THE PLANE AT WHICH THE INTERSECTION OF A PLANE AND A CONICAL SURFACE DEFINES A DEGENERATE ELLIPSE AND HYPERBOLA;
- recognize each conic, given the locus definition;
- describe, orally and in writing, the conic, given the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed HORIZONTAL OR VERTICAL line;
- generate the graph of quadratic relations with the use of graphing calculators or graphing utility packages;
- describe, orally and in writing, and identify the conic defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$;
- DESCRIBE, ORALLY AND IN WRITING, AND IDENTIFY THE EFFECT ON THE VALUE OF B IN THE EQUATION OF A CONIC IN THE FORM $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, WHEN THE GRAPH OF THE CONIC IS ROTATED FROM ITS POSITION WHEN $B = 0$;
- describe, orally and in writing, and identify the conic formed when given the value of the eccentricity;
- describe, orally and in writing, and identify the eccentricity when given the conic;
- describe, orally and in writing, and identify the conic formed when given the locus definition;
- identify and graph the conic when given A POINT ON THE CONIC, a fixed point, and the eccentricity;
- calculate the eccentricity when given a fixed HORIZONTAL OR VERTICAL line, a fixed point, and a point on the conic;
- identify and graph the conic when given the eccentricity, a fixed point, and a fixed HORIZONTAL OR VERTICAL line;
- describe, orally and in writing, and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when TWO of the numerical coefficients change;
- participate in and contribute toward the problem-solving process for problems that require the analysis of quadratic relations studied in Mathematics 30.

Students demonstrating excellent achievement can also:

- describe, orally and in writing, the combination of values for the numerical coefficients of the general quadratic relation that would result in the degenerate conics;
- use the locus definition to verify the equation of each conic section;
- describe, orally and in writing, and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when TWO or more of the numerical coefficients change;
- describe, orally and in writing, and identify the changes in the graph of a conic when the eccentricity changes;
- DESCRIBE ORALLY, IN WRITING, AND BY MODELING, AND IDENTIFY THE POSITION OF THE PLANE AT WHICH THE INTERSECTION OF A PLANE AND A CONICAL SURFACE DEFINES A DEGENERATE PARABOLA;
- complete the solution to problems that require the analysis of quadratic relations studied in Mathematics 30.

The student demonstrating an acceptable achievement can do the following types of questions, as can the student demonstrating excellent achievement.

1. The conic represented by $2x^2 + x - 3y - 25 = 0$ is

   A. a circle
   * B. a parabola
   C. an ellipse
   D. a hyperbola

2. A conic is represented by $3x^2 + 4y^2 + 5x + Ey - 36 = 0$. Predict and then describe the changes that will happen to the graph when +5 is changed to –4 and –36 is changed to –9.

3. A conic is described as having an eccentricity of 2. This conic is

   A. a circle
   B. a parabola
   C. an ellipse
   * D. a hyperbola

4. The orbit of a comet has an eccentricity of 1.3. Describe the path that this orbit is following.

5. A conical surface is intersected by a plane that is parallel to its generator. The shape of the conic section formed is

   A. a circle
   * B. a parabola
   C. an ellipse
   D. a hyperbola

6. An object moves along a path such that the sum of the distances from two fixed points is constant. The path of the object can be described as

   A. a circle
   * B. an ellipse
   C. a parabola
   D. a hyperbola

7. Find an equation of a horizontal or vertical directrix that corresponds to a conic with eccentricity of $\frac{1}{2}$, a point on the conic of (–2,3) and a focus of (–4, 0). [4 possible solutions: $y = 3 \pm 2\sqrt{13}$ OR $x = 2 \pm 2\sqrt{13}$]*

8. Halley's Comet has a period of 76 years, that is, Halley's Comet is seen once every 76 years. The orbit of Halley's Comet has an eccentricity of 0.96. Sketch the graph of the orbit.

9. A plane intersects a conical surface. The plane is perpendicular to its axis and passes through the vertex. The locus produced by the intersection of the conical surface and the plane is

A. a line
* B. a point
C. a circle
D. an ellipse

10. Using a computer, Michele and Robin graphed a quadratic relation defined by $25x^2 + 16y^2 + 30x - 400 = 0$. Now they wish to graph a circle. Which value should Michele and Robin increase to graph the circle?

A. The coefficient of x^2
* B. The coefficient of y^2
C. The coefficient of x
D. The constant term

11. Describe the effect on an ellipse as the cutting plane approaches the vertex of the conical surface.

The student demonstrating excellent achievement can do questions such as:

1. A conic is represented by $Ax^2 + Cy^2 + 3x + Ey - 36 = 0$. Predict and then describe what happens to the graph of the conic when 3 is changed to –4 and –36 is changed to –9.

2. The equation $Dx + F = 0$ is the **complete** equation of a degenerate

A. circle
* B. parabola
C. ellipse
D. hyperbola

3. As the focus moves closer to the centre of an ellipse, describe the effect on the eccentricity.

4. Given a fixed line and a fixed point, describe the location of a point on a conic as the eccentricity changes.

5. Yui Lin is sketching the graph of a conic. First she plots a point P at (6,0). Then she plots two other points on the conic, Q (9,4) and R (9, −4). What additional information does Yui Lin need in order to know whether the conic will be a hyperbola or a parabola?

A. The line $y = 0$ is the axis of symmetry
B. The line $x = 6$ is tangent to the curve
* C. The domain of the relation
D. The range of the relation

6. The equation $Ax^2 - Cy^2 = 0$ is the equation of a degenerate

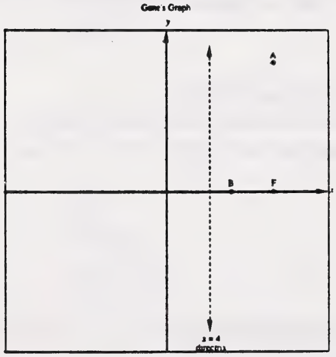
A. circle
B. parabola
C. ellipse
* D. hyperbola

The student demonstrating acceptable achievement as well as the student demonstrating excellent achievement can *completely draw the conic* described in this written-response question.

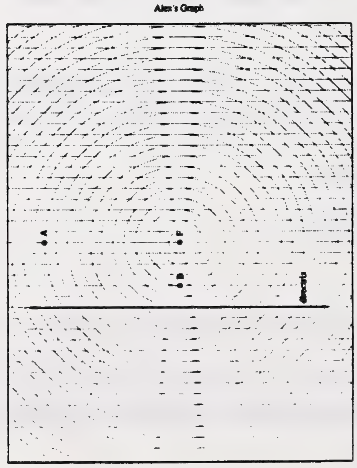
7. Before class ended, Gene and Alex started drawing the graph of the same conic. Both drew the fixed line (directrix) and plotted a fixed point (focus). Just as the bell rang to end the class, Gene and Alex plotted two other points, A and B, on the conic. Below, on the Cartesian plane, is Gene's graph. On the following page, drawn on the circle line grid, is Alex's graph.

Complete either Gene's or Alex's graph. Show how you decided which conic the students were drawing.

Gene's Graph



Alex's Graph



Exponential and Logarithmic Functions

Students can describe the relationship between exponential and logarithmic functions.

Students demonstrating acceptable achievement can:

- generate the graph of exponential and logarithmic functions with the use of graphing calculators or graphing utility packages;
- recognize and sketch the graphs of exponential and logarithmic functions and recognize their inverse relationship;
- convert functions from exponential form to logarithmic form and vice versa;
- apply the laws and properties of logarithms to evaluate logarithmic expressions;
- solve AND VERIFY simple exponential and logarithmic equations;
- state the domain and range of the exponential and logarithmic functions;
- use the graphs of the exponential and logarithmic functions to estimate the value of one of the variables, given the other variable;
- participate in and contribute toward the problem-solving process for problems that can be represented by logarithmic or exponential functions studied in Mathematics 30.

Students demonstrating excellent achievement can also:

- solve AND VERIFY exponential and logarithmic equations;
- complete the solution to problems that can be represented by logarithmic or exponential functions studied in Mathematics 30.

The student demonstrating an acceptable achievement can do the following types of questions, as can the student demonstrating excellent achievement.

1. An equivalent form of $\frac{3}{4}\log_7(x) = 5$ is

   A. $x^4 = 7^{15}$
   B. $x^3 = 5^{28}$
   C. $x^3 = 20^7$
   * D. $x^3 = 7^{20}$

2. The range of $f(x) = 2^x$ is

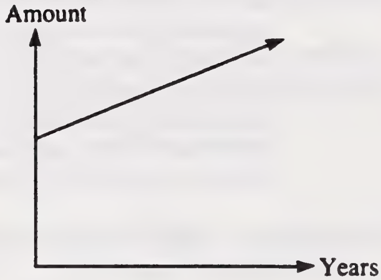
   A. $x \geq 0$
   B. $x > 0$
   C. $f(x) \geq 0$
   * D. $f(x) > 0$

3. If $4^{2x} = 90$, then the value of x correct to the nearest tenth is ________[1.6]*.

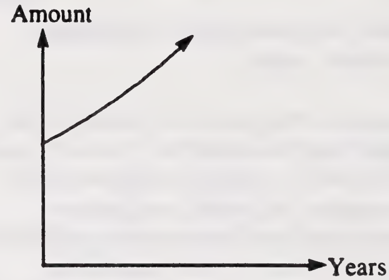
4. If $2^x = 8$, then the value of x correct to the nearest tenth is ___[3.0]*.

5. A radioactive substance decays exponentially so that after four years, half of its original amount remains. The graph that best represents this relation is

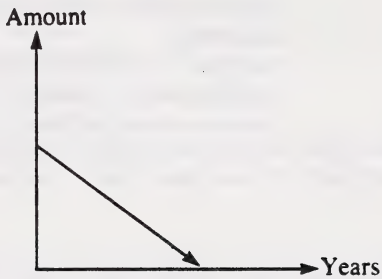
A.



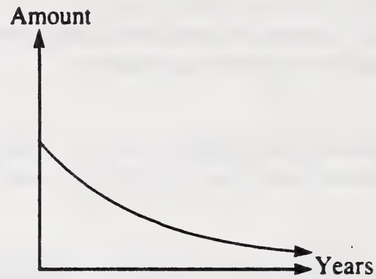
B.



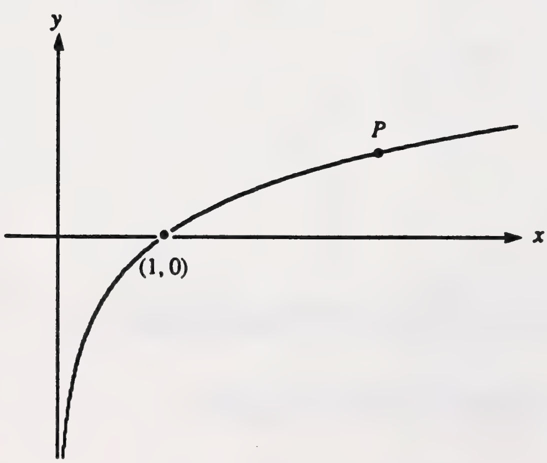
C.



D.



6. The sketch of the graph of $y = \log_2(x)$ is shown below. If $P(x, 1.54)$ is a point on this graph, then the value of x correct to the nearest hundredth is________ [2.91]*.



The student demonstrating excellent achievement can do questions such as:

1. If $2\log_{10}(x) + \log_{10}(y) = 3$ and $3\log_{10}(x) - \log_{10}(y) = 7$, then x and y respectively are

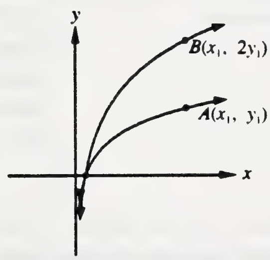
   A. 10 and 10
   B. 10 and 0.1
   * C. 100 and 0.1
   D. 100 and 10

2. If $3^{2x-5} = 5^{x+1}$, then the value of x correct to the nearest tenth is ___[12.1]*.

3. If $\log_{(x-4)}(x^2 - 2x - 61) = 2$, then the value of x correct to the nearest tenth is ___[12.8]*.

4. In the diagram at the right, A is on the graph of $y = \log_7(x)$ and B is on the graph of

   A. $y = \log_7(2x)$
   B. $y = \log_7(x^{1/2})$
   * C. $y = \log_7(x^2)$
   D. $y = (\log_7 x)^2$



The student demonstrating acceptable or excellent achievement can *complete* questions such as this written-response question:

5. For the equation $\log_5(x-4) + \log_5(x-2) = \log_5(3)$, find the value of x.

The student demonstrating excellent achievement can also *complete* a question such as:

6. For the equation $\log_5(x-4) + \log_5(x-2) = 3$, find the value of x.

Permutations and Combinations

*All students can describe the difference between a permutation and a combination and calculate the number of permutations or combinations of **n** things taken **r** at a time AND APPLY THESE TO THE EXPANSION OF BINOMIALS.*

Students demonstrating acceptable achievement can:

- calculate the number of LINEAR, CIRCLE, AND RING permutations AND PERMUTATIONS WITH REPETITIONS of n things taken r at a time;
- calculate the number of combinations of n things taken r at a time;
- expand binomials of the form $(x + a)^n$, $n \in W$ using the Binomial Theorem;
- describe, orally and in writing, the difference between a permutation and a combination;
- participate in and contribute toward the problem-solving process for problems involving permutations AND/or combinations, including probability problems studied in Mathematics 30.

Students demonstrating excellent achievement can also:

- EXPAND BINOMIALS OF THE FORM $(x + By)^n$, $n \in W$ USING THE BINOMIAL THEOREM AND DETERMINE SPECIFIC TERMS OF THIS EXPANSION;
- EXPLAIN THE REASON WHY THERE ARE DIFFERENT NUMBERS OF PERMUTATIONS WHEN A GIVEN NUMBER OF OBJECTS ARE ARRANGED IN A LINE, A CIRCLE, OR IN A RING, OR WHEN SOME OF THE OBJECTS ARE REPEATED OR IDENTICAL;
- complete the solution to problems involving permutations AND/or combinations, including probability problems studied in Mathematics 30.

The student demonstrating an acceptable achievement can do the following types of questions, as can the student demonstrating excellent achievement.

1. In how many ways can six different mathematics books be arranged on a shelf? [6!]*

2. In how many ways can a committee of four members be selected from a 10 member student council? $[{}_{10}C_4]$*

3. In how many ways can seven girls stand in a row if Marissa has to be in the centre? [6!]*

4. In how many ways may Francis make his choice if he is allowed to choose seven out of nine questions on an examination? $[{}_9C_7]$*

5. Find the coefficient of the x^3 term in the expansion of $(x + 2)^5$. [40]*

6. In the expansion of $(x + y)^7$, how is the coefficient determined in the term containing $x^6 y$? What is the value of the coefficient? [7]*

7. In the expansion of $(x + y)^7$, how many terms are there? How does this relate to the exponent of the binomial? [8,n +1]*

8. What is the probability of getting two heads and one tail if three coins are tossed once? $[\frac{3}{8}]$*

9. In how many different orders can five people sit at a round table? [4!]*

10. Concert organizers are determining the order in which the school bands from Fort McMurray, Grande Prairie, Lethbridge, Red Deer, and Medicine Hat will perform. The number of ways to arrange the order in which the bands will perform if Red Deer performs first is

A. 4
B. 20
* C. 24
D. 120

11. Martha wants to use the digits 2, 3, 4, or 5 for her personal banking identification number. If repetitions are allowed, how many different four-digit numbers can she create?

* A. 256
B. 120
C. 24
D. 16

12. Find the number of possible arrangements of all the letters in the word *curriculum*. $[\frac{10!}{2!2!3!}]$*

The student demonstrating excellent achievement can do questions such as:

1. A Mathematics 30 class was asked to find how many selections of five fruits can be made from five peaches, four pears, two apples, and one grapefruit. What, if any, assumptions must students make when doing this problem? Explain. Do students have enough information to solve this problem?

2. In how many different orders can five people sit at a round table if Jack and Jill must sit next to one another? [12]*

3. Find the coefficient of the x^3y^4 term in the expansion of $(x-2y)^7$. [560]*

4. How many different arrangements of five letters are possible if two letters are chosen from the word *down* and three letters are chosen from the word *blue*? $[{}_4C_3 \bullet {}_4C_2 \bullet 5!]$*

5. Three men and three women are planning to sit at a round table. The group decides on a seating plan that alternates man-woman-man-woman-man-woman. How many such arrangements are possible?

   A. 6

   * B. 12

   C. 18

   D. 36

The student demonstrating acceptable achievement can *complete parts a and b* in this written-response question, whereas the student demonstrating excellent achievement can *complete all three parts.*

6. For the high school basketball game, 4 cheerleaders are working on a special routine.

   a. In how many ways can the 4 cheerleaders arrange themselves in a row?

   b. In how many ways can the 4 cheerleaders arrange themselves in a circle?

   c. Explain why there are more ways for the 4 cheerleaders to arrange themselves in a row than in a circle.

Sequences and Series

Students can describe the differences between sequences and series with an emphasis on arithmetic and geometric sequences, terms of arithmetic and geometric sequences, and can determine the sums of arithmetic and geometric series.

Students demonstrating acceptable achievement can:

- WRITE THE SPECIFIC TERMS OF A SEQUENCE GIVEN ITS DEFINING FUNCTION;
- expand a series given in sigma notation;
- describe, orally and in writing, the difference between sequences and series, arithmetic or geometric, INFINITE AND FINITE;
- apply the general term formula for arithmetic and geometric sequences;
- apply the sum formula for arithmetic and geometric series;
- participate in and contribute toward the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

Students demonstrating excellent achievement can also:

- solve problems using the general term and/or sum formulas in which there are two unknowns;
- WRITE THE SPECIFIC TERMS OF A SEQUENCE GIVEN ITS RECURSIVE DEFINITION;
- DETERMINE THE FUNCTIONS THAT DESCRIBE ANY SEQUENCE THAT HAS A RECOGNIZABLE PATTERN;
- complete the solution to problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

The student demonstrating an acceptable achievement can do the following types of questions, as can the student demonstrating excellent achievement.

1. If the sum of the first 16 terms of an arithmetic series is 40 and the common difference is 5, then the first term of this series is

   A. −9
   * B. −35
   C. −38
   D. −70

2. The value of $\sum_{n=3}^{6} (-2)^n$ is

   * A. 40
   B. 42
   C. 120
   D. 126

3. In a geometric sequence, $a = 125$ and $t_4 = 13\,824$. Correct to the nearest tenth, the common ratio for this sequence is________[4.8]*.

4. During each 25-year period, an isotope of strontium has its initial mass reduced by a factor of $\frac{1}{2}$. The initial mass of a sample of strontium is 36 mg. The mass of this sample after 325 years is

A. 0.0022 mg
* B. 0.0044 mg
C. 0.0088 mg
D. 0.0176 mg

The student demonstrating excellent achievement can do questions such as:

1. In an arithmetic sequence, $t_4 + t_{13} = 99$ and $t_7 = 39$. The first term of this sequence is

A. −7
* B. −3
C. 3
D. 7

2. The nth term of a series is given by $t_n = 5n - 3$. An expression for the sum of n terms of this series is

A. $S_n = \frac{5}{2}(n^2 - n)$

B. $S_n = \frac{5}{2}n^2 - n$

C. $S_n = \frac{5n^2 + n}{2}$

* D. $S_n = \frac{5n^2 - n}{2}$

3. A culture of bacteria is being studied in a genetics experiment. The researcher observes that the bacteria double in number every 15 min. After 8 h, the number of bacteria in the culture is N. At this rate, how long will it take for the total population of bacteria to reach $16N$?

* A. 9 h

B. $9\frac{1}{4}$ h

C. 16 h

D. 128 h

4. Given the sequence $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, . . . determine the defining function for the nth term.

$[f(n) = \frac{n}{n+1}]$*

The student demonstrating acceptable achievement can *complete* both these written-response questions, as can the student demonstrating excellent achievement.

5. An auditorium has eight seats in the first row. Each subsequent row has four more seats than the preceding row.

   a. How many seats are there in the 16th row?

   b. All together, there are 1400 seats in the auditorium. How many rows of seats are there?

6. a. Find the nth term, t_n, of a sequence where the first term, t_1, is 6; the second term, t_2, is 12; and the third term, t_3, is 24.

   b. A sequence different from the one in part (a) has a first term, t_1, of 6 and a third term, t_3, of 24. Find the nth term, t_n.

The student demonstrating excellent achievement can also *complete* this written-response question:

7. Can the number 525 be written as the sum of consecutive numbers? If so, find the sequence. Is there more than one sequence? If so, find the other sequences.

Appendix D

Explanation of Levels

Procedures

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate;
- give reasons for the steps in a procedure;
- reliably and efficiently execute procedures;
- verify the results of procedures empirically (e.g., using models) or analytically;
- recognize correct and incorrect procedures;
- generate new procedures and extend or modify familiar ones;
- appreciate the nature and role of procedures in mathematics.

It is important that students know how to execute mathematical procedures reliably and efficiently; a knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they have given a correct answer; they also must understand concepts underlying a procedure and the logic that justifies it. Procedural knowledge also involves the ability to differentiate those procedures that work from those that do not and the ability to modify them or create new ones. Students must be encouraged to appreciate the nature and role of procedures in mathematics, that is, they should appreciate that procedures are created or generated as tools to meet specific needs in an efficient manner and thus can be extended or modified to fit new situations. The assessment of students' procedural knowledge, therefore, should not be limited to an evaluation of their facility in performing procedures; it should emphasize all the aspects of procedural knowledge addressed in this standard.

Concepts

The assessment of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts;
- identify and generate examples and nonexamples;
- use models, diagrams, and symbols to represent concepts;
- translate from one mode of representation to another;
- recognize the various meanings and interpretations of concepts;
- identify properties of a given concept and recognize conditions that determine a particular concept;
- compare and contrast concepts.

In addition, assessment should provide evidence of the extent to which students have integrated their knowledge of various concepts.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of common examples; it encompasses the broad range of abilities identified in this standard. Assessment, too, must address these aspects of conceptual understanding. Assessment tasks should focus on students' abilities to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and nonexamples, to represent concepts in various ways, and to recognize students' various meanings. Tasks that ask students to apply information about a given concept in novel situations provide strong evidence of students' knowledge and understanding of that concept. Problems designed to elicit information about students' misconceptions can provide information useful in planning or modifying instruction.

Problem Solving

The assessment of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems;
- apply a variety of strategies to solve problems;
- solve problems;
- verify and interpret results;
- generalize solutions.

Students' ability to solve problems develops over time as a result of extended instruction, opportunities to solve many kinds of problems, and encounters with real-world situations. Students' progress should be assessed systematically, deliberately, and continually to effectively influence their confidence and ability to solve problems in various contexts. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments also should yield evidence of students' use of strategies and problem-solving techniques and of their ability to verify and interpret results. Finally, because the power of mathematics is derived, in part, from its generalizability, this aspect of problem solving should be assessed as well.

From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 209, 223, 228.

Appendix E

Guidelines for Significant Digits, Manipulation of Data, and Rounding in the Mathematics, Chemistry, and Physics Diploma Examinations

Significant Digits

1. For all nonlogarithmic values, regardless of decimal position, any of the digits 1 to 9 is a significant digit; 0 may be significant. For example:

123 0.123 0.00230 2.30×10^3
all have 3 significant digits

2. Leading zeros are not significant. For example:

0.12 and 0.012 have two significant digits

3. Trailing zeros to the right of the decimal are significant. For example:

0.123 00 and 20.000 have five significant digits

4. Zeros to the right of a whole number are considered to be ambiguous. **The Student Evaluation Branch considers all trailing zeros to be significant**. For example:

200 has three significant digits

5. For logarithmic values, such as pH, any digit to the left of the decimal is **not** significant. For example:

a pH of 1.23 has two significant digits, but a pH of 7 has no significant digits

Manipulation of Data

1. When adding or subtracting measured quantities, the calculated answer should be rounded to the same degree of precision as that of the least precise number used in the computation **if this is the only operation**. For example:

$$\begin{array}{rl} 12.3 & \text{(least precise)} \\ 0.12 & \\ \underline{12.34} & \\ 24.76 & \end{array}$$

The answer should be rounded to 24.8.

2. When multiplying or dividing measured quantities, the calculated answer should be rounded to the same number of significant digits as are contained in the quantity with the fewest number of significant digits **if this is the only operation**. For example:

(1.23)(54.321) – 66.81483

The answer should be rounded to 66.8.

3. When a series of calculations is performed, the answer should not be rounded off based upon interim values. For example:

(1.23)(4.321)/(3.45 - 3.21) = 22.145125

The answer should be rounded to 22.1.

Rounding

1. When the first digit to be dropped is less than or equal to 4, the last digit retained should not be changed. For example:

1.2345 rounded to three digits is 1.23

2. When the first digit to be dropped is greater than or equal to 5, the last digit retained should be increased by one. For example:

12.25 rounded to three digits is 12.3